

# Workshop on Dynamics Numeration and Tilings (FloripaDynSys)

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**SBM**



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# Computability obstructions in multidimensional symbolic dynamics

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While the theory of one-dimensional subshift of finite type is linked with finite automata, the study in higher dimensions has connections with computability theory. For example, even the basic problem which consist to decide if a subshift of finite type is empty becomes undecidable in dimension 2. During a long time this undecidability appears only as obstructions in the study of dynamical properties but recent results use computability concepts to describe it. Thus local constraints can imply "complex" dynamical properties in the computability setting. This talk explore these recent connexions between dynamical properties and computability concepts.

# Topological and measurable full groups

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The notion of full group was introduced by H. Dye in his study of orbit equivalence of measured dynamical systems. It was then extended to the Borel and to the topological case. In this sequence of talks I will review the known results both in the measurable and the Borel case before studying the topological case. To  $(X, \phi)$  is a minimal homeomorphism of the Cantor set (i.e.,  $(X, \phi)$  is a Cantor minimal system) is associated two different full groups; the so-called finite full group or topological full group  $\mathbb{F}(\phi)$  and the full group  $\mathbb{F}(\phi)$ .

We will review their main properties and in particular describe the recent new results on the topological full group.

# Topological models in holomorphic dynamics

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In this mini-course we will describe topological models for dynamical systems in one and several complex variables. In particular we will focus on the family of the so-called complex Hénon mappings which exhibits a great variety of behaviours. Our goal will be to understand the topological properties of the various attractors that appear naturally in such dynamical systems.

# Dynamics, Fractal Geometry and Number Theory: the Lagrange spectrum

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The study of good approximations of real numbers by rational numbers is a classic subject of Number Theory, related to several other important subjects, as Diophantine equations and the theory of transcendence. One of the fundamental facts about diophantine approximations is that every irrational number can be approximated by rational numbers  $p/q$  with error  $|\alpha - p/q| < 0$  for which there are infinitely many rational numbers  $p/q$  such that  $|\alpha - p/q|$

# Asymptotically periodic piecewise contractions of the interval

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Let  $A_1, \dots, A_n$  be a sequence of pairwise disjoint compact subintervals of  $(0,1)$ . Let  $\phi_i: [0,1] \rightarrow A_i$  be a  $C^2$ -diffeomorphism such that  $\sup_{x \in (0,1)} |\phi_i(x)| \rightarrow 0$



# Invariant measures for nonprimitive adic transformations

Fisher Albert <sup>1</sup>

1 : Marina Talet  
*Univer Aix-Marseille*

We present a classification of the invariant measures for Vershik's adic transformations. We address the nonstationary, nonprimitive case, where interesting measures which are infinite on every open subset can occur. The Bratteli diagrams are assumed to have a bounded number of vertices and the measures are finite on some subdiagram defined by erasing vertices and edges. Our results extend theorems of Bezuglyi, Kwiatkowski, Medynets and Solomyak.

# Ergodic properties of random iterations of skew products

Cirilo Patricia<sup>1</sup>

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We will prove ergodic properties for random skew products on the cylinder, obtaining a version of Kakutani's theorem for such systems.

More specifically, we study systems of the form  $(\omega, y) \mapsto (\sigma\omega, \Phi_{\omega_0}(y))$  where  $\sigma$  is the `\textit{shift}` on the symbolic space  $\Omega$  and  $y$  belongs to the cylinder  $\mathbb{S}^1 \times \mathbb{R}$ . We prove that, when the skew product is conservative, then it is ergodic if and only if the maps  $\Phi_i$  have no common non-trivial invariant set. It is also true that these systems are rationally ergodic with return sequence asymptotic to  $\sqrt{n}$ . This is a joint work with Yuri Lima and Enrique Pujals.

# Skew-composed flows

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Skew-composed flows is a particular kind of systems which include quasiperiodically forced-circle homeomorphisms, homeomorphisms of generalized solenoids and other kind of almost-periodic behaviour. We will show in this talk how these flows allow for an easy formulation of Poincar'e-like theorem ensuring that the flow is actually semiconjugated to a translation. This joint work with T. Jäger

# **Instability in celestial mechanics: the $N+2$ centre problem and weak forms of hyperbolicity.**

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We will discuss a particular problem in celestial mechanics which can be consider as a first approximation to the  $N+2$  body problem where 2 of the bodies move faster than the others. We will try to explain how the Partially Hyperbolic theory gives some insights in understanding the solution of that celestial mechanic configuration.

# **A thermodynamic formalism for continuous time Markov chains with values on the Bernoulli space: entropy, pressure and large deviations**

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We analyze the ergodic properties of continuous time Markov chains with values on the Bernoulli space. We consider as the infinitesimal generator the operator  $L=L_A - I$ , where  $L_A$  is a discrete time Ruelle operator (transfer operator), and  $A$  is a given fixed Lipschitz function defined on the Bernoulli space. The associated continuous time stationary Markov chain will define the a priori probability.

Given a Lipschitz interaction  $V$ , we are interested in Gibbs (equilibrium) state for such  $V$ . This will be another continuous time stationary Markov chain. In order to analyze this problem we will use a continuous time Ruelle operator (transfer operator) naturally associated to  $V$ . Among other things we will show that a continuous time Perron-Frobenius Theorem is true in the case  $V$  is a Lipschitz function.

We also introduce an entropy, which is negative, and we consider a variational principle of pressure. Finally, we analyze large deviations properties for the empirical measure in the continuous time setting using results by Y. Kifer

# Dynamical systems of type $(m,n)$

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Given positive integers  $n$  and  $m$ , we consider dynamical systems in which (the disjoint union of)  $n$  copies of a topological space is homeomorphic to  $m$  copies of that same space. The universal such system is shown to arise naturally from the study of a  $C^*$ -algebra denoted by  $O_{m,n}$ , which in turn is obtained as a quotient of the well known Leavitt  $C^*$ -algebra  $L_{m,n}$ , a process meant to transform the generating set of partial isometries of  $L_{m,n}$  into a tame set. Describing  $O_{m,n}$  as the crossed product of the universal  $(m,n)$ -dynamical system by a partial action of the free group  $F_{m+n}$ , we show that  $O_{m,n}$  is not exact when  $n$  and  $m$  are both greater than or equal to 2, but the corresponding reduced crossed product, denoted by  $O_{m,n}^r$ , is shown to be exact and non-nuclear. Still under the assumption that  $m, n \geq 2$ , we prove that the partial action of  $F_{m+n}$  is topologically free. This is joint work with Pere Ara from the Universitat Autònoma de Barcelona.

# Ergodic Optimization and Phase Transitions in Shift Spaces

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We discuss some results about maximizing measures (existence and support properties) and the connection of those facts with the analyticity of the Pressure in some countable Markov shifts. We also study some natural questions from the Ergodic Optimization point of view in the context of Statistical Mechanics (Ising Models) in multidimensional shift spaces. The results are part from two works in progress with Ricardo Freire (IME-USP) and Leandro Cioletti (Unb-Brazil).

# Structural theory of transition classes

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Given a finite-to-one factor code  $\pi$  from a one dimensional shift of finite type  $X$  onto a sofic shift  $Y$  there is a well-known quantity assigned to  $\pi$  called the degree of the code and is defined to be the minimal number of preimages of points of  $Y$ . When  $\pi: X \rightarrow Y$  is infinite-to-one, a notion analogous to the degree of a finite-to-one code, called the class degree, was defined recently. The class degree is the minimal number of transition equivalence classes over points of  $Y$  where the definition of transition classes is motivated by communicating classes in Markov chains. We describe the structure of such transition classes and show that the class degree which is a generalization of the degree satisfies similar properties. Joint work with S. Hong and U. Jung.



# Examples of $C^*$ -algebras inspired by higher dimensional symbolic dynamic

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$C^*$ -algebras, inspired by the non-commutativity of quantum theory, are natural non-commutative candidates generalizing a commutative space  $C_0(X)$  with an action. For instance, Cuntz and Krieger recovered the Bowen--Franks invariant of a shift of finite type by computing the extension groups and  $K$ -theory of their so-called Cuntz--Krieger  $C^*$ -algebras associated to shifts of finite type.

In this talk we discuss some examples of Cuntz--Krieger type algebras inspired by higher dimensional symbolic dynamical systems, that is, systems with an action by  $\mathbb{Z}^2$  or an amenable group. We obtain some uniqueness and  $K$ -theory results for these algebras.

# The Frenkel-Kontorova model for almost-periodic environments

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In this talk, we discuss the existence of minimizing configurations associated to generalized Frenkel-Kontorova models on quasi-crystals. This is a joint work with Samue Petite (Université de Picardie) and Philippe Thieullen (Université de Bordeaux).

# The Dynamics of Inverse Semigroups

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Inverse semigroups are semigroups such that each element  $s$  has a unique "inverse"; that is, an element  $t$  such that  $sts = s$  and  $tst = t$ . Actions of inverse semigroups on spaces may be studied as semigroup actions, but it turns out that each inverse semigroup  $S$  gives rise to a topological space on which  $S$  acts intrinsically. Hence, inverse semigroups are inherently dynamical objects, despite being defined only algebraically. The action of  $S$  on this intrinsic space give rise to an étale groupoid and hence a  $C^*$ -algebra. We give some examples, specifically the polycyclic monoids (which give rise to the one-sided full shift and the Cuntz algebras), self-similar groups, and self-similar graph actions. In the latter cases, we show that in many cases the  $C^*$ -algebra is isomorphic to a partial crossed product of the intrinsic space by a group. This is joint work with Ruy Exel.

# Prime numbers, determinism and pseudorandomness

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The difficulty of the transition from the representation of an integer in a number system to its multiplicative representation (as a product of prime factors) is at the source of many important open problems in mathematics and computer science.

We will present a survey on recent results concerning the study of independence between the multiplicative properties of integers and various "deterministic function", i. e. function produced by a dynamical system of zero entropy or defined using a simple algorithm, in connection with the Chowla and Sarnak conjectures on Mobius randomness principle

# A class of equal-entropy extensions of multidimensional full shifts

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We present a class of  $\mathbb{Z}^2$  subshifts of finite type given by nearest neighbor rules (i. e. transition matrices) which have topological entropies of the form  $\log N$  for a fixed but arbitrary natural number  $N \in \mathbb{N}$ . We then show that those matrix shifts always factor onto the  $\mathbb{Z}^2$  full shift of the same entropy (we study the explicitly given factor maps and their properties) and that they can be constructed to realize all known (uniform) mixing properties.

# Relative class degrees and multiplicative class degree inequality

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Class degrees of codes on shifts of finite type were devised extending the notion of degrees of finite-to-one codes on shifts of finite type. Many of their properties have been revealed to be nice analogues of degrees of finite-to-one codes, though, multiplicative equality for degrees of compositions of finite-to-one codes is broken for class degrees of compositions of, possibly infinite-to-one, codes in general. We introduce relative class degrees and show that multiplicative inequality holds instead for class degrees.

# A construction of strictly ergodic subshifts having entropy dimension

Jung Uijin <sup>1\*</sup>

1 : Ajou University

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The entropy dimension of shift spaces (indeed, that of any topological dynamical systems) was introduced to measure a subexponential growth rates of the number of blocks. Any positive entropy subshift has entropy dimension 1, and any Sturmian or substitutional subshift has entropy dimension 0. In this talk, for any  $h$  in  $[0,1]$ , we give a construction of a strictly ergodic subshift of entropy dimension  $h$ . This is a joint work with K. K. Park and J. Lee.

# Equicontinuous actions and full groups

Cortéz Maria <sup>1\*</sup>

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In this talk we give a dynamical way to characterize isomorphic full groups for free equicontinuous minimal actions on the Cantor sets. This is a joint work in progress with Kostya Medynets.



# Phase-transitions in the quadratic family

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In this talk we show the first examples of a low-temperature phase-transition in the quadratic family for the geometric pressure function in the real and complex case. We show examples of a first-order and a high-order phase transition. In both examples, the dynamics is transitive and the parameters are Collet-Eckmann.

# Spectra of Stochastic Adding Machines

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This is a Joint Work with Ali Messaoudi.

Adding one to a non-negative integer  $n$  can be performed through an algorithm that changes digits one by one in the expansion of  $n$  on some system of numeration. Stochastic adding machines are time-homogeneous Markov Chains on the non-negative integers that are built based on a stochastic rule that prevents the algorithm to finish. Killeen and Taylor have introduced this concept considering dyadic expansions. They found out and studied in detail an interesting relation between complex dynamics and the spectral properties of the transition operators of a particular class of dyadic stochastic machines: the spectra of the transition operators were filled-in Julia sets of degree two polynomials. Thereafter stochastic machines have been studied on several other systems of numerations. This generates a rich class of examples connecting probability theory, operator theory and complex dynamics.

Here we are going to define stochastic adding machines based on Cantor Systems of numeration. Then we compute the parts of the spectra of the transition operators associated to these stochastic machines in different Banach spaces as  $C_0, C$  and  $L_\alpha, 1 \leq \alpha \leq +\infty$ .

# A bridge between Büchi automata and graph directed iterated function systems

Leroy Julien<sup>1</sup>

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In this talk, I will present a joint work with Emilie Charlier and Michel Rigo. We obtained an equivalence between sets of real numbers recognizable by some finite Büchi automaton and sets of real numbers that are the attractor of some graph directed iterated function system. This allows us to prove a conjecture of Adamczewski and Bell about an analogue of Cobham's theorem for reals.

# Reducible substitutions: The geometry of cobounds/ height

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We look at the notion of "height of a constant length substitution" (compare Dekking '78), respectively "cobounds of a primitive substitution" (compare Ferenczi-Mauduit-Nogueira '96) from a geometrical and number-theoretic viewpoint. This allows us to better understand the calculation of the spectrum of the associated dynamical systems, and also see under which condition we can reduce the height. Appropriate examples of substitutions, and one- and two-dimensional substitution tilings will be given.

# Geometrical representation for a class of symbolic dynamical systems

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Let  $A$  be an alphabet over  $d$  letters and  $A^*$  be the set of finite and non-empty words over  $A$ . A substitution

$\sigma$  is a map from  $A$  to  $A^*$ . It is known that to any substitution we can associate a shift symbolic dynamical system.

In this work we will consider a class  $\mathcal{C}$  of substitutions over an alphabet of 3 letters. The interest of this class remains in the fact that the dynamical system associated to a substitution  $\sigma$  in  $\mathcal{C}$  is measure theoretically isomorphic to an exchange of pieces over a compact set  $\mathcal{K}_\sigma$  of  $\mathbb{R}^2$  (called Rauzy fractal). The points where the metrics isomorphism is not one to one corresponds to the boundary of  $\mathcal{K}_\sigma$ . We will prove some geometrical and dynamical properties of this boundary.

# Eigenvalues of minimal Cantor systems

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In this talk we will review some results characterizing continuous and non-continuous eigenfunctions of the Koopman operator. We will focus on minimal dynamical systems with finite topological rank.

# A class of Cubic Rauzy Fractal

Rodrigues Tatiana<sup>1</sup>

1 : Universidade Estadual Paulista Júlio de Mesquita - UNESP (BRAZIL)

This is a joint work with Jefferson Bastos, Ali Messaoudi and Daniel Smania.

In this paper, we study arithmetical and topological properties of a class of Rauzy fractals given by the polynomial  $x^3 - ax^2 + x - 1$  where  $a > 2$  is an integer. We give explicitly an automaton that generates the boundary of the elements of this class. Using this automaton, we show the number of neighbours of the Rauzy fractal (in the periodic tiling) is 8. We also prove that the boundary of Rauzy fractal is homeomorphic to a topological circle.

# Symbolic Dynamics associated to Substitution Tilings

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It is known that some tilings has representations using symbolic dynamics. This is done in order to compute the Cohomology and  $K$ -Theory associated to a Tiling. This kind of representation is often associated with one-dimensional substitution tilings. In this poster, the author presents an interpretation that can be applied to specific examples of two-dimensional tilings.



# Enveloping semigroups of systems of order $d$

Donoso Sebastián<sup>1</sup>

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We study the Ellis semigroup of a  $d$ -step nilsystem and the inverse limit of such systems. By using the machinery of cubes developed by Host, Kra and Maass, we prove that such a system has a  $d$ -step topologically nilpotent enveloping semigroup. In the case  $d=2$ , we prove that these notions are equivalent, extending a previous result by Glasner.

# Direct Hamiltonization - The generalization of an alternative Hamiltonization

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A new procedimento named direct Hamiltonization gives another foundations to Hamiltonian Analytical Mechanics, since in this formalism the Hamiltonian function can be obtained for any mechanical system. The main change proposed in this procedimento is that the conjugate momenta cannot be defined a priori, but instead of this, they are determinate as a consequence of a canonical description of the mechanical system. As the direct Hamiltonization contains the alternative one, then the usual Hamiltonization and momenta is recovered while the envelope solution is selected. Also this procedimento assures the existence of a Hamiltonian function without any constraints whatsoever mechanical system is considered, therefore the usual quantization is always allowed. There are an infinity of possible extensions and applications of these procedure, as in field theory or dynamical systems.

# Stochastic Adding Machine and 2-Dimensional Julia Sets

Asmat Uceda Rafael, Messaoudi Ali <sup>1\*</sup>

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In this work, we define a stochastic adding machine associated to a quadratic base  $(F_n)_{(n \geq 0)}$  and we study its spectral and topological properties. In particular we prove that the spectrum of the transition operator associated to its transition matrix is connected to the filled Julia set of a quadratic map of  $\mathbb{C}^2$ .

# Subshifts of Finite Type

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We intend to show some preliminar results about a kind of subshift of finite type.

# Julia set of an holomorphic correspondence

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We introduce the definition of Julia set of the holomorphic  $(w-c)^q=z^p$  and also give an estimate of its Hausdorff dimension using some tools of Thermodynamic Formalism.

# The Poincaré Recurrence Theorem

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The work is essentially the study of the Poincaré Recurrence Theorem along with the necessary knowledge for its understanding and proof.

The Poincaré recurrence theorem states that given a finite measure space  $(X, A, m)$  and a Measure-Preserving Transformation  $T$  from  $X$  to  $X$ , then for every  $E$  in the sigma-algebra  $A$  we have that almost every point of  $E$  returns an infinite number of times to  $E$  through  $T$ .

This theorem consists of a first result of ergodic theory which is an important tool in the areas of Dynamical Systems and Statistical Mechanics, as well as being an interesting application of Measure Theory which focuses on generalize the notions of length, area and volume using functions known as measures

# **A version of Mozes' theorem for limit shifts of two-dimensional substitutions**

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In this work we present a proof that every limit subshift generated by a  $2 \times 2$  substitution is of sofic type. To do this we construct an explicit extension by using as infrastructure a minimal subshift which is a subset of the Robinson tiling and which provides a hierarchical skeleton for all points.

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